

PHOTON RADIATION IN A HEAT BATH

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Abstract

We discuss the bremsstrahlung of photons into a heat bath, and calculate from first principles the energy radiated. Even to lowest order the spectrum of the radiation at low frequency is no more singular than at zero temperature. In addition to the obvious contributions, this spectrum includes terms associated with fluctuations.

In a recent paper, Weldon^[1] has considered the spectrum of photons emitted when a charged particle scatters off a potential within a heat bath. As he observes, in such a problem infrared effects are more delicate than at zero temperature, because in addition to the usual factors the integrands involved contain Bose distributions

$$f(k^0) = \frac{1}{e^{|k^0|/T} - 1} \quad (1)$$

which diverge at zero energy. According to Weldon, while the infrared divergences do nevertheless still cancel in a lowest-order perturbative calculation (this is an extension of the zero-temperature Bloch-Nordsieck mechanism), they leave behind a spectrum of emitted photons whose number density $dN/d\omega$ at zero-frequency ω is unacceptable: it behaves as ω^{-2} , so that the energy density $\omega dN/d\omega$ behaves as ω^{-1} , which integrates to give a divergent result for the total energy carried off by the photons*.

Weldon further shows how to sum the infrared-sensitive terms to all orders. In this note, we are not concerned with this, the most important part of Weldon's beautiful paper. Rather, we seek to clarify his remarks about the lowest-order calculation of the radiated energy. We do so within the framework of a very simple model^[2], which we believe incorporates the essential features. This is the decay, within a heat bath that contains only photons, of a scalar uncharged particle P into a pair of scalar charged particles ("muons"). We calculate three different quantities: (I) the order- e^2 contribution $\Delta\Gamma$ to the total decay rate; (II) the probability that the muons emerge with combined centre-of-mass energy within ΔE of the parent-particle energy; and (III) the net energy spectrum $\omega dN/d\omega$ transferred to the heat bath by the photons.

For (I), we agree with Weldon that $\Delta\Gamma$ is finite. For (II), we find that the probability in order e^2 behaves as $(\Delta E)^{-1}$; we expect that Weldon's resummation procedure would remove this singular behaviour. It is for (III) that we differ from Weldon: $\omega dN/d\omega$ is finite at $\omega = 0$ even to order e^2 .

As is well-known, the decay rate may be calculated from a discontinuity of a thermal Green's function^[3]. However, it appears that the energy spectrum $\omega dN/d\omega$ cannot be calculated in terms of such a discontinuity. Instead, we must introduce a different method, which we derive from first principles. This method reproduces the same result for the decay rate as the conventional one, and may be used also for calculating $\omega dN/d\omega$.

We recall first that in finite-temperature gauge theory one may choose that at bare-propagator level only the physical degrees of freedom are thermalised^[4]. Then the thermal part of the photon propagator

$$\Delta_T(k) = 2\pi\delta(k^2)f(k) \quad (2a)$$

* See equation (1.7) of reference 1

represents the effects of real photons in the heat bath. This is easy to understand. The finite-temperature photon propagator is, in any gauge,

$$Z^{-1} \text{tr} e^{-H_0/T} T A(x) A(0) = Z^{-1} \text{tr} e^{-H_0/T} (\langle 0 | T A(x) A(0) | 0 \rangle + : A(x) A(0) :) \quad (3a)$$

where

$$Z = \text{tr} e^{-H_0/T} \quad (3b)$$

and the traces are defined as summation over expectation values in only *physical* states. The first term in (3a) is just the zero-temperature Feynman propagator in whatever gauge is being used, and is unaltered by the heat bath. The second receives contributions only from the real photons in the heat bath and is

$$Z^{-1} \text{tr} e^{-H_0/T} \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta^+(k^2) \frac{d^4 k'}{(2\pi)^4} 2\pi \delta^+(k'^2) \left\{ a^\dagger(k) a(k') e^{ik \cdot x} + a^\dagger(k') a(k) e^{-ik \cdot x} \right\} \quad (2b)$$

which is the Fourier transform of (2a).

We return now to the decay in the heat bath of the uncharged particle into two muons. We have previously^[2] calculated, to lowest order e^2 in the muon charge, the change $\Delta\Gamma$ in the decay rate caused by the heat bath. This change involves only the thermal part (2) of the photon propagator. It consists of three pieces: (i) a term that may be interpreted as the acquisition by the muons of an additional mass of order eT , (ii) another that may be interpreted as a shift induced by the heat bath in the coupling λ that governs the decay, and (iii) a term that corresponds to stimulated emission of photons into the heat bath or absorption from it. We found that the mass shift (i) is

$$\delta m^2 = 2e^2 \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^2) f(k^0) \quad (4a)$$

and the terms (ii) and (iii) contribute

$$2P^0 \Delta\Gamma = \lambda^2 e^2 \frac{1}{(2\pi)^5} \int d^4 k \delta(k^2) f(k^0) d^4 p_1 d^4 p_2 \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \left[- \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 \right] \\ \{ -\delta^4(p_1 + p_2 - P) + \delta^4(p_1 + p_2 + k - P) \} \quad (4b)$$

where in the last factor the first δ -function corresponds to the term (ii) and the second to (iii). We have supposed that the decaying particle has momentum P .

In the integrals (2), k denotes photon 4-momentum. Because only the thermal part (2) of the photon propagator is involved, the photon is real, even when its effect is interpreted as just changing the value of the coupling λ , or of the muon mass. These two changes together correspond to the absorption of a real photon from the heat bath and its re-emission with the same momentum. The amplitude for this is the sum of the order- e^2 graphs shown in figure 1a and 1b, where the dashed lines at the bottom are the spectator photons in the heat bath. Both of the active photons in the graphs must have the same momentum k , because to obtain an order- e^2 contribution to the decay rate we must multiply the sum of graphs in figures 1a and 1b by the order- e^0 graph of figure 1c, in which all the heat-bath photons are spectators. This product corresponds to the first two of the cut graphs shown in figure 2, in which the photon line represents the thermal part (2a) of the photon propagator. Because this thermal propagator contains $\delta(k^2)$, rather than $\delta^+(k^2)$, each single cut graph of figure 2 corresponds to two contributions, as indicated in figures 1a and 1b. The first δ -function in the curly bracket in (4b) arises from the first two cut graphs in figure 2, while the second δ -function arises from the other two cut graphs. The latter represent the sum of the squares of the terms shown in figure 1d and 1e.

In his calculation of the net photon energy spectrum $\omega dN/d\omega$, Weldon appears not to have included the contributions of figure 1a, 1b and 1e. We maintain that it is necessary to do so if one wishes to calculate a physically-meaningful quantity. In order to construct such a quantity, we must carefully perform a thought experiment. We shall consider two such experiments. In both, we think of the heat bath as a microwave oven with only photons in it. Any muons that are produced as a result of decays

escape straight through the oven walls. Our world has no other charged particles, in particular no electrons.

In our first thought experiment, we drill a small hole in one of the oven walls and imagine observing the energy spectrum $\omega dN/d\omega$ of radiation emitted through it, just as one does in the usual analysis of black-body radiation^[5]. We suppose that we have introduced into the oven a collection of the particles P that decay into two muons, and consider the spectrum of photons in events triggered by the decay-product muons. We calculate the difference between this photon spectrum at temperature T and that at zero temperature.

In order to do this calculation, we go back to first principles. Define single-photon states r , initially discrete with energies ϵ_r , and label the states of the heat bath by the corresponding occupation numbers n_r . Then, with a decaying particle P also in the oven, the initial density matrix is

$$\rho_i = Z^{-1} \sum_{\{n\}} \exp\left(-\sum_r n_r \epsilon_r / T\right) |n_1, n_2, \dots, P\rangle \langle n_1, n_2, \dots, P| \quad (5a)$$

with

$$Z = \prod_r \frac{1}{1 - e^{-\epsilon_r/T}} \quad (5b)$$

After a long time, the density matrix has become

$$\rho_f = Z^{-1} \sum_{\{n\}} \exp\left(-\sum_r n_r \epsilon_r / T\right) \mathcal{T} |n_1, n_2, \dots, P\rangle \langle n_1, n_2, \dots, P| \mathcal{T}^\dagger \quad (6a)$$

Here, as usual, \mathcal{T} is the interaction part of the S -matrix, $S = 1 + i\mathcal{T}$. Because our initial state contains an unstable particle, which inevitably decays, we know that an interaction takes place. The decay rate of the particle P is calculated from the expectation value of ρ_f in the states

$$|m_1, m_2, \dots, \mu(p_1)\mu(p_2)\rangle \quad (6b)$$

with summation over the photon occupation numbers m_1, m_2, \dots and over the momenta p_1 and p_2 of the decay-product muons. For this, we need the photon-operator structure of the \mathcal{T} -matrix. The various matrix elements in figure 1 correspond to the following photon-operator terms in \mathcal{T} :

$$e^2 \sum_s A_s a_s^\dagger a_s + e^2 \sum_s B_s a_s a_s^\dagger + C + e \sum_s D_s a_s^\dagger + e \sum_s E_s a_s \quad (7)$$

The operators A_s, B_s, \dots correspond to figures 1a,b, \dots and involve the photon modes s only through conserving energy and momentum. They convert the initial particle P into the two muons and include also the internal muon propagators. We use the familiar properties $a|n\rangle = \sqrt{n} |n-1\rangle$ and $a^\dagger|n\rangle = \sqrt{n+1} |n+1\rangle$, so that the order- e^2 contribution to the summed expectation value takes the form

$$\Delta\Gamma = Z^{-1} \sum_{\{n\}} \exp\left(-\sum_r n_r \epsilon_r / T\right) \sum_{s, p_1, p_2} \left(n_s P_s^{AC} + (n_s + 1) P_s^{BC} + (n_s + 1) P_s^{DD} + n_s P_s^{EE} \right) \quad (8a)$$

Here

$$P_s^{AC} = e^2 \langle \mu(p_1)\mu(p_2) | A_s | P \rangle \langle P | C^\dagger | \mu(p_1)\mu(p_2) \rangle + e^2 \langle \mu(p_1)\mu(p_2) | C | P \rangle \langle P | A_s^\dagger | \mu(p_1)\mu(p_2) \rangle \quad (8b)$$

and similarly for P_s^{BC}, P_s^{DD} and P_s^{EE} . Notice that, in this order- e^2 expression for $\Delta\Gamma$, the DD and EE terms are the squares of order- e amplitudes, while the AC and BC terms are the interference between order- e^2 and order- e^0 amplitudes. The presence of the interference terms at finite temperature is possible because the order- e^0 amplitude in figure 1c involves spectator photons in the heat bath. One of these spectator photons has to be identified with the photon that is absorbed and emitted in the diagrams of figure 1a and 1b. For this to be possible, the absorbed and emitted photon must

have the same momentum as the corresponding spectator photon in figure 1c, and so the photon scattering in figures 1a and 1b is forward scattering.

We use

$$\bar{n} \equiv Z^{-1} \sum_n n \epsilon^{-n\epsilon/T} = f(\epsilon) \quad (9)$$

where $f(\epsilon)$ is the Bose distribution (1), so that (8a) is

$$\Delta\Gamma = \sum_{s,p_1,p_2} (f(\epsilon_s)P_s^{AC} + (f(\epsilon_s) + 1)P_s^{BC} + (f(\epsilon_s) + 1)P_s^{DD} + f(\epsilon_s)P_s^{EE}) \quad (10)$$

Subtracting off the zero-temperature limit, we retrieve in the continuum limit the formula (4b).

We now calculate the thermal average of the number of photons in state t . This comes from the expectation value of $a_t^\dagger a_t \rho_f$ in the states (6b), again with summation over the photon occupation numbers m_1, m_2, \dots and over the momenta p_1 and p_2 of the decay-product muons. Because ρ_f is not normalised to unity, we must introduce also a factor Γ^{-1} :

$$\begin{aligned} \langle N_t \rangle = \Gamma^{-1} Z^{-1} \sum_{\{n\}} \exp\left(-\sum_r n_r \epsilon_r / T\right) \sum_{p_1, p_2} \Big\{ & n_t^2 P_t^{AC} + n_t(n_t + 1)P_t^{BC} + (n_t + 1)^2 P_t^{DD} + (n_t - 1)n_t P_t^{EE} \\ & + n_t P^{CC} + n_t \sum_{s \neq t} (n_s P_s^{AC} + (n_s + 1)P_s^{BC} + (n_s + 1)P_s^{DD} + n_s P_s^{EE}) \Big\} \end{aligned} \quad (11)$$

We use

$$\overline{n^2} \equiv Z^{-1} \sum_n n^2 \epsilon^{-n\epsilon/T} = \bar{n}^2 + f(\epsilon)(f(\epsilon) + 1) \quad (12)$$

so that (11) is

$$\langle N_t \rangle = f(\epsilon_t) + F^{\text{emission}}(\epsilon_t) + F^{\text{absorption}}(\epsilon_t) + F^{\text{fluc}}(\epsilon_t) \quad (13a)$$

where

$$\begin{aligned} F^{\text{emission}}(\epsilon_t) &= \Gamma^{-1} \sum_{p_1, p_2} (f(\epsilon_t) + 1) P_t^{DD} \\ F^{\text{absorption}}(\epsilon_t) &= -\Gamma^{-1} \sum_{p_1, p_2} f(\epsilon_t) P_t^{EE} \\ F^{\text{fluc}}(\epsilon_t) &= \Gamma^{-1} \sum_{p_1, p_2} f(\epsilon_t)(f(\epsilon_t) + 1) [P_t^{AC} + P_t^{BC} + P_t^{DD} + P_t^{EE}] \end{aligned} \quad (13b)$$

Evidently, the first term in (13a) is the value $\langle N_t \rangle$ would have if there was no decaying particle in the heat bath. We subtract it off. As expected, the emission term, corresponding to part of the square of figure 1d, adds to the spectrum, while the absorption term, corresponding to part of the square of figure 1e, has negative sign and so reduces it. From (12), the term $F^{\text{fluc}}(\epsilon_t)$, because it involves $f(f + 1)$, is associated with fluctuations in the heat bath.

Subtracting off the zero-temperature limit, and going over to the continuum limit, we are left with the change in the spectrum per decaying particle:

$$2P^0 \Gamma \frac{d}{d\omega} \Delta N(\omega) = \Delta^{\text{emission}} + \Delta^{\text{absorption}} + \Delta^{\text{fluc}} \quad (14a)$$

with

$$\begin{aligned} \Delta^{\text{emission}} + \Delta^{\text{absorption}} &= f(\omega) \frac{\lambda^2 e^2}{(2\pi)^5} \int d^4 k \delta(k^0 - \omega) \delta(k^2) d^4 p_1 d^4 p_2 \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \\ &\quad J^2 \left\{ \delta^4(p_1 + p_2 + k - P) - \delta^4(p_1 + p_2 - k - P) \right\} \end{aligned} \quad (14b)$$

and

$$\begin{aligned} \Delta^{\text{fluc}} = & f(\omega)(f(\omega) + 1) \frac{\lambda^2 e^2}{(2\pi)^5} \int d^4 k \delta(k^0 - \omega) \delta(k^2) d^4 p_1 d^4 p_2 \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \\ & J^2 \left\{ \delta^4(p_1 + p_2 + k - P) + \delta^4(p_1 + p_2 - k - P) - 2\delta^4(p_1 + p_2 - P) \right\} \\ & + f(\omega)(f(\omega) + 1) \frac{4\lambda^2 e^2}{(2\pi)^5} \int d^4 k \delta(k^0 - \omega) \delta(k^2) \frac{\partial}{\partial m^2} \int d^4 p_1 d^4 p_2 \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \delta^4(p_1 + p_2 - P) \end{aligned} \quad (14c)$$

where

$$J^2 = \left[- \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 \right] \quad (14d)$$

The last term in (14c) is the part of the interference between figures 1a and 1b with figure 1c that is associated with the thermal mass shift δm^2 of (4a).

When $\omega \rightarrow 0$, $\frac{d}{d\omega} \Delta N(\omega)$ diverges as ω^{-1} . The ω^{-2} mentioned by Weldon^[1] is present in $F^{\text{absorption}}$ and F^{emission} , but it cancels between them and leaves behind ω^{-1} . The separate contributions to the fluctuation term behave as ω^{-3} . An important component of the fluctuation term is the interference between the diagrams of figures 1a and 1b with figure 1c; the δm^2 term is a part of this interference term^[2]. It is straightforward to see that the ω^{-3} behaviour cancels between the various parts of the fluctuation term, as does its ω^{-2} behaviour. Moreover, one can show that, no matter how fast the decaying particle is moving (or not moving) before it decays, the ω^{-1} behaviour cancels also; this cancellation is closely related to the cancellation that occurs at low temperature in $\Delta\Gamma$, making^[2] it behave as T^4 instead of T^2 . As we show in the Appendix, it is a result of current conservation together with unitarity. So the fluctuation term is nonsingular when $\omega \rightarrow 0$; it actually goes to zero like ω . However, notice that the ω^{-1} behaviour in the fluctuation term cancels only because we have integrated over the momenta p_1 and p_2 of the final-state charged particles. In the scattering problem considered by Weldon^[1], p_1 rather is the momentum of an incoming particle and is not integrated, so we believe that then the cancellation of the ω^{-1} contributions to the fluctuation terms does not take place.

Thus the breakdown of perturbation theory signalled by Weldon in fact does not occur for the measurement he considered. However, it does occur in certain other cases. To see this, consider our second thought experiment (II), where instead of measuring the number density of the photons emitted from the heat bath we measure the total energy of the two muons. We consider the rate at which the pair of muons emerges with total energy within magnitude ΔE of the energy of their parent particle. This is calculated from (4b) with an additional

$$\theta(\Delta E - |P^0 - p_1^0 - p_2^0|) \quad (15)$$

under the integral. In spite of the presence in (4b) of the factor that is quadratically divergent at $k = 0$, the k -integration is convergent, because although the difference of δ -functions in the last factor is linear in k , the rest of the integrand is even in k . However, although this cancellation of divergences takes place between the two δ -functions, as $\Delta E \rightarrow 0$ the answer is again divergent. The term with the first δ -function clearly is independent of ΔE , while for small ΔE the second behaves as ΔE^{-1} . This behaviour is unphysical (it is even large negative) and so must be corrected by a resummation such as was suggested by Weldon^[1]. Of course, the same resummation is necessary for this quantity at zero temperature, though then before the resummation the divergence is only as $\log \Delta E$.

We have explained that our method of calculation requires us to go back to first principles. It gives the same answer for the decay rate as the conventional approach, but may be applied also to the calculation of the photon energy spectrum $\omega dN/d\omega$ or the number spectrum $dN/d\omega$. While the decay rate is linear in the Bose distribution $f(\omega)$, the number spectrum $dN/d\omega$ contains a quadratic term, which is associated with fluctuations in the heat bath. Therefore, it does not seem to be associated with a discontinuity of a thermal Green's function. Our initial attempts to calculate $dN/d\omega$ from the

discontinuity diagrams of figure 2 yielded an absorption term whose sign was opposite from that in (13b), which is clearly incorrect; as we have shown, the correct sign is obtained only when one takes account of the fact that part of the absorption effect is a contribution to the fluctuation term.

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Appendix

In this Appendix, we calculate the leading terms in the photon spectrum (14) for small ω . We verify that the ω^{-2} terms cancel in the emission and absorption parts of (14b), leaving ω^{-1} . We find further that there is a cancellation between the different parts of the fluctuation term (14c), so that while the separate terms behave as ω^{-3} their sum behaves as ω . This is closely related to the cancellation, found in [2], of the low-temperature contributions to $\Delta\Gamma$. As we explain at the end of this Appendix, it is a consequence of current conservation and unitarity.

In order to deal with (14), we follow reference 2 and make the changes of variables

$$p_1 \rightarrow p_1 \mp \frac{p_2 \cdot k}{P \cdot k} k, \quad p_2 \rightarrow p_2 \mp \frac{p_1 \cdot k}{P \cdot k} k \quad (A1)$$

so that

$$J^2 \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \delta^4(p_1 + p_2 - P \pm k) \longrightarrow J^2 \delta^+(p_1^2 - m^2 \mp X) \delta^+(p_2^2 - m^2 \mp X) \delta^4(p_1 + p_2 - P) \quad (A2)$$

where

$$X = \frac{2p_1 \cdot k \, p_2 \cdot k}{P \cdot k} \quad (A3)$$

We then expand (A2) in powers of X , to the second order for the fluctuation contribution to (14), and to first order for the emission and absorption contributions. The result is

$$\Delta^{\text{emission}} + \Delta^{\text{absorption}} \sim f(\omega) \frac{\lambda^2 e^2}{(2\pi)^5} \int d^4 k \delta(k^2) \delta(k^0 - \omega) A \quad (A4a)$$

and

$$\Delta^{\text{fluc}} \sim f(\omega)(f(\omega) + 1) \frac{\lambda^2 e^2}{(2\pi)^5} \int d^4 k \delta(k^2) \delta(k^0 - \omega) B \quad (A4b)$$

where

$$A = \int d^4 p_1 d^4 p_2 \delta^4(P - p_1 - p_2) (-2X J^2) (\partial/\partial m^2) \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) \quad (A5a)$$

and

$$B = \int d^4 p_1 d^4 p_2 \delta^4(P - p_1 - p_2) [X^2 J^2 (\partial/\partial m^2)^2 + 4(\partial/\partial m^2)] \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2), \quad (A5b)$$

with J^2 defined in (14d).

Evidently both A and B are Lorentz invariants and so they can depend upon k via the invariant $P \cdot k$ only. Therefore we can first calculate with the P -particle at rest (relative to the heat bath), and then replace $M\omega$ by $P \cdot k$ to get the result for a moving P -particle. For P at rest, and with θ the angle between \mathbf{k} and \mathbf{p}_1 ,

$$\frac{1}{2} \int_{-1}^1 d(\cos \theta) X J^2 = \frac{2M}{\omega} \left[1 - \frac{2m^2}{M\sqrt{(M^2 - 4m^2)}} \ln \left(\frac{M + \sqrt{(M^2 - 4m^2)}}{M - \sqrt{(M^2 - 4m^2)}} \right) \right] \quad (A6a)$$

$$\frac{1}{2} \int_{-1}^1 dx X^2 J^2 = \frac{2}{3} (M^2 - 4m^2) \quad (\text{A6b})$$

We have anticipated that we are going to insert these integrals into (A5) and (A4), and so set $k^2 = 0$ and $p_1^2 = m^2 = p_2^2$. We require also

$$\int d^4 p_1 d^4 p_2 \delta^4(P - p_1 - p_2) \delta^+(p_1^2 - m^2) \delta^+(p_2^2 - m^2) = \pi \frac{\sqrt{(M^2 - 4m^2)}}{2M} \quad (\text{A6c})$$

Since (A6b) and (A6c) do not depend on ω , they remain the same when the decaying particle P is not at rest in the heat bath. Using (A6) we find that the two derivatives in the expression (A5b) for B give equal and opposite contributions when they are inserted into (A4b). This cancellation is exactly similar to that which occurs in the change $\Delta\Gamma$ induced by the heat bath in the low-temperature decay rate^[2].

Thus Δ^{fluc} vanishes in this approximation: it receives its first contribution from the X^4 terms in the expansion of (A2) and so behaves as $\omega^3 f(\omega)(f(\omega) + 1)$ at low frequency. The leading behaviour of $\frac{d}{d\omega} \Delta N(\omega)$ near $\omega = 0$ comes from the emission and absorption terms and is, from (A5a) and (A6a),

$$2P^0 \Gamma \frac{d\Delta N}{d\omega} \sim \frac{2\lambda^2 e^2}{(2\pi)^3} \frac{f(\omega)}{\sqrt{(P^0)^2 - M^2}} \ln \left(\frac{M + \sqrt{(M^2 - 4m^2)}}{M - \sqrt{(M^2 - 4m^2)}} \right) \ln \left(\frac{P^0 + \sqrt{(P^0)^2 - M^2}}{P^0 - \sqrt{(P^0)^2 - M^2}} \right) \quad (\text{A7})$$

where we have used

$$\int d^4 k \delta(k^0 - \omega) \delta(k^2) (P \cdot k)^{-1} = \frac{2\pi}{\sqrt{(P^0)^2 - M^2}} \ln \left(\frac{P^0 + \sqrt{(P^0)^2 - M^2}}{P^0 - \sqrt{(P^0)^2 - M^2}} \right)$$

The reason for the cancellation in $\Delta\Gamma$, and of the leading terms in B , is the following. $\Delta\Gamma$ is calculated from the trace of the final-state density matrix ρ_f given in (6a). Through unitarity, this is

$$\begin{aligned} \text{tr } Z^{-1} \sum_{\{n\}} \exp \left(- \sum_r n_r \epsilon_r / T \right) \mathcal{T} |n_1, n_2, \dots, P\rangle \langle n_1, n_2, \dots, P| \mathcal{T}^\dagger \\ = -i Z^{-1} \sum_{\{n\}} \exp \left(- \sum_r n_r \epsilon_r / T \right) \langle n_1, n_2, \dots, P | (\mathcal{T} - \mathcal{T}^\dagger) | n_1, n_2, \dots, P \rangle \end{aligned} \quad (\text{A8})$$

Because we are working to lowest order in the charge e , only one photon in the state $|n_1, n_2, \dots, P\rangle$ is active, with the other photons just providing the number factors n_s in (8a), so that in the continuum limit $\Delta\Gamma$ is an integral, involving the Bose factor f , over the photon momentum k of the imaginary part of the scattering amplitude $\langle P, k^\mu | \mathcal{T} | P, k^\nu \rangle$. Because the electromagnetic current is conserved, for off-shell k this amplitude has the usual decomposition:

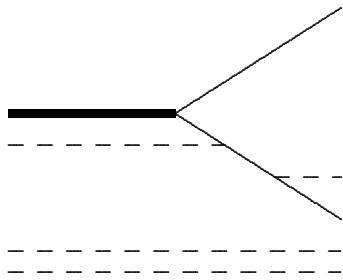
$$-\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) T_1 + \left(P^\mu - \frac{P \cdot k}{k^2} k^\mu\right) \left(P^\nu - \frac{P \cdot k}{k^2} k^\nu\right) T_2 \quad (\text{A9})$$

This has to be contracted with transverse real-photon polarisation vectors and k^2 set to 0. This eliminates T_2 . But when $k^2 \rightarrow 0$, $T_1 \sim (P \cdot k)^2 T_2 / k^2$, and T_2 vanishes like k^2 . Because we do not allow the unstable particle P to survive in the final state, the amplitudes T_1 and T_2 do not have poles corresponding to this particle, and so T_1 behaves as $(P \cdot k)^2$ for small $P \cdot k$. It is this that makes $\Delta\Gamma$ behave^[2] as T^4 at low temperature T . According to (8a), when the zero-temperature limit has been subtracted off, $\Delta\Gamma$ is calculated from the combination $[P_t^{AC} + P_t^{BC} + P_t^{DD} + P_t^{EE}]$. This same combination appears in the fluctuation term (13b) and the $(P \cdot k)^2$ there gives the additional factor ω^2 .

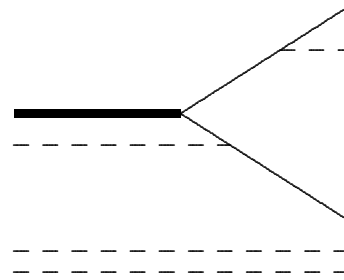
Note that it is essential to this argument that we have integrated over the momenta p_1 and p_2 of the final-state muons.

References

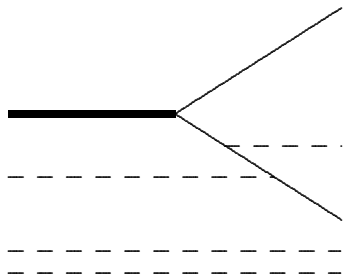
- 1 H A Weldon, West Virginia preprint WVU-932
- 2 M Jacob and P V Landshoff, Physics Letters B281 (1992) 114
- 3 H A Weldon, Physical Review D42 (1990) 2384; R Kobes and G W Semenoff, Nuclear Physics B272 (1986) 329; N Ashida et al, Physical Review D45 (1992) 2066
- 4 P V Landshoff and A Rebhan, Nuclear Physics B383 (1992) 607
- 5 eg F Mandl, *Statistical Physics*, Wiley 1971



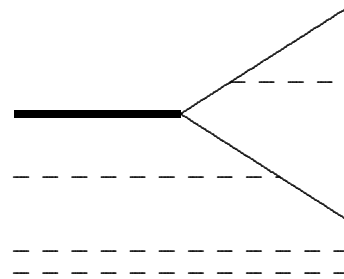
(a)



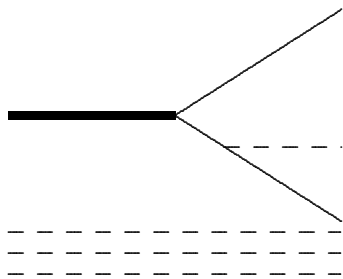
(b)



(c)



(d)



(e)

Figure 1

Photon radiation in a heat bath, involving both active and spectator photons.

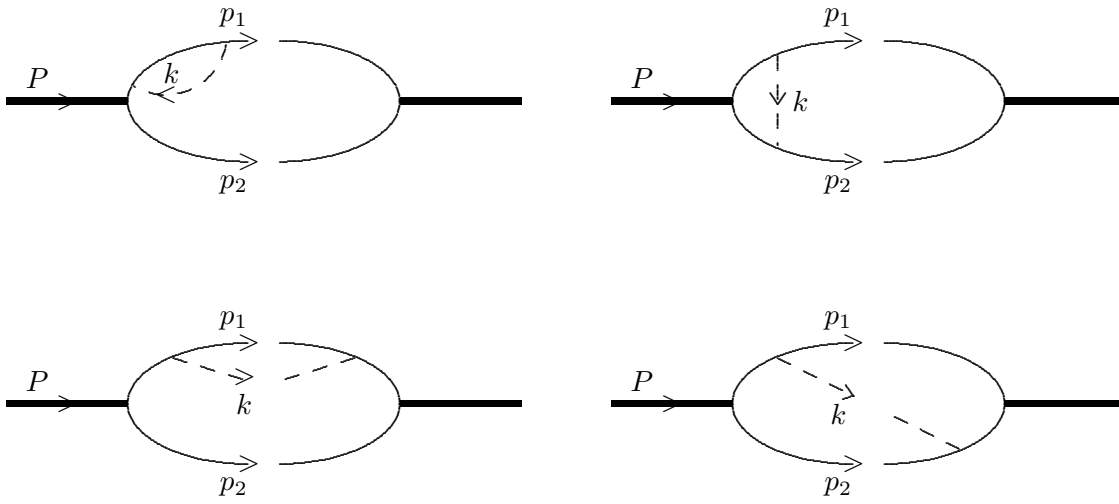


Figure 2

Cut diagrams for the square of the amplitude corresponding to the various terms of figure 1